Journal of Pure and Applied Mathematics: Advances and Applications
Volume 12, Number 1, 2014, Pages 77-104

OPTIMAL CONTROL OF NIPAH VIRUS (NIV) INFECTIONS: A BANGLADESH SCENARIO

M. H. A. BISWAS

Mathematics Discipline
Science Engineering and Technology School
Khulna University
Khulna-9208
Bangladesh
e-mail: mhabiswas@yahoo.com

Abstract

Nipah virus (NiV) is a newly detected highly pathogenic virus with ability to cause devastating morbidity and mortality (an estimated 100% in some cases) rate among the human populations. This emerging infectious disease has become one of the most alarming threats of the public healths in Bangladesh mainly due to its periodic outbreaks (as it strikes almost every year) and the highly devastating mortality rate. In this paper, we propose a mathematical model describing the host-pathogen interactions in terms of ordinary differential equations (ODEs). The main aim is to investigate the disease propagation and control strategy of NiV infections. The behaviour of the dynamics of NiV infections has been illustrated by the numerical simulations.

1. Introduction

Bangladesh has been the most risky geographic distribution for several epidemic and infections diseases like the newly detected deadly Nipah virus (NiV) infections in the south-east region of Asia [23]. The

2010 Mathematics Subject Classification: Primary: 92D30, 49K15; Secondary: 34A34.
Keywords and phrases: Nipah virus, infections disease, mathematical model, numerical analysis, control strategy.
Received June 13, 2014

very recently occurred and even the periodical outbreaks of Nipah virus infections indicate the serious alarming and devastating threats of the public healths in Bangladesh as well in the world. The outbreaks of NiV infections in Bangladesh are assumed to be the most alarming and thus the significantly different in epidemiologic and clinical features [9] because of the fact that they have been occurring every year (except 2002 and 2006) since the first detection of Nipah virus and its devastating infections in Bangladesh in 2001. 11 outbreaks have already occurred from the years 2001 to 2013 (until 6 April, 2013), causing 167 deaths among the identified 209 as seriously infected by NiV with the average mortality rate of 80%. A statistics of the chronological outbreaks of NiV infections and the increasing average mortality rate is shown in Table 1. From the Table 1, we see that only in the years 2011 and 2012, the mortality rate is 100%, which is of course a great threat for the global public healths along with Bangladesh. However, steps for proper treatments and control strategies should be taken immediately right now. Although significant numbers of research have been carried out individually and/or jointly in the Institute of Epidemiology, Disease Control and Research (IEDCR) and the International Center for Diarrhoeal Disease Research, Bangladesh (ICDDR,B) with the other national and international collaborations on this deadly virus and mentionable research works have been published in the internationally reputed journals, the world health expertise should pay special attention to end this highly fatal disease for ever from Bangladesh. Otherwise, it may pose a significant threat to global health if the outbreaks become more widespread with an average mortality rate of 79%, since this is a virus that is devastating to the families affected. We refer readers to ([2], [6], [14], [16], [21], [22], [23] and the references within) for more detailed study on NiV infections and to ([3], [4], [5], [20]) for some recent developments of other communicable diseases.

In this paper, we attempt to propose a dynamic model of NiV infections and discuss its possible control and preventive strategy via optimal control techniques. Dynamic models of 'NiV'-like *zoonotic* viruses and their evolutions were discussed in [1]. In the vein of [1], Biswas in [2] (see also [6]) proposed a mathematical model of NiV infections in the form of "SIR"-type epidemic model and discussed its possible transmissions in context of Bangladesh. The aim of this study is mainly to extend the NiV model proposed in [2] by introducing two control variables in the model and thus reformulate the dynamical model of NiV infections in terms of ordinary differential equations. We solve the model numerically and then analyze the behaviour of the disease dynamics. A control strategy is proposed by the illustrations of numerical simulations. Before going to in-depth analysis of the model, it is worth presenting a brief discussions on NiV infections and disease transmissions in Bangladesh for the readers convenient.

2. NiV and its Outbreaks in Bangladesh

Nipah virus (NiV), of the family *Paramyxoviridae* and the genus *Henipavirus*, is a *zoonotic* (as it is transmitted from animals to humans) virus that causes outbreaks of fatal encephalitis in humans [9]. The human Nipah virus (NiV) infection was first recognized in a large outbreak of 276 reported cases in peninsular Malaysia and Singapore from September 1998 through May 1999 (see, for example, [8], [10], [11], [18], and [24]). The virus was first isolated from a patient from Sungai Nipah village in Malaysia and the name 'Nipah' was first introduced according to the name of that village. Most of the cases presented primarily with encephalitis and mortality rate of 39%, had close contact with sick pigs [25], which indicates that the host of NiV infections in 1998 at Malaysia outbreaks was the pigs. However, large fruit bats of the genus *Pteropus* appear to be the natural reservoir of NiV and the pigs are assumed to be infected from those fruit bats. The possible ways of how the pigs might be infected from the fruit bats in Malaysia outbreaks were discussed in [18].

NiV was first detected in Bangladesh in 2001. It was also identified in India for the first time in 2001 and second time in 2007 (see Figure 2). Unfortunately, 11 (eleven) outbreaks have already occurred in Bangladesh since the first detection of NiV in 2001, with highly mortality rate an estimated 80% in an average and 100% in some cases (see Table 1). The most alarming fact is that almost every year in winter (December to March), the deadly NiV strikes in the north and western regions of Bangladesh. Until at the end of 2008, about 14 districts of Bangladesh were affected by NiV outbreaks (see Figure 1), which at the end of 2013, have been expanded to more than 22 districts of north-western and central regions of Bangladesh (see Figure 2).

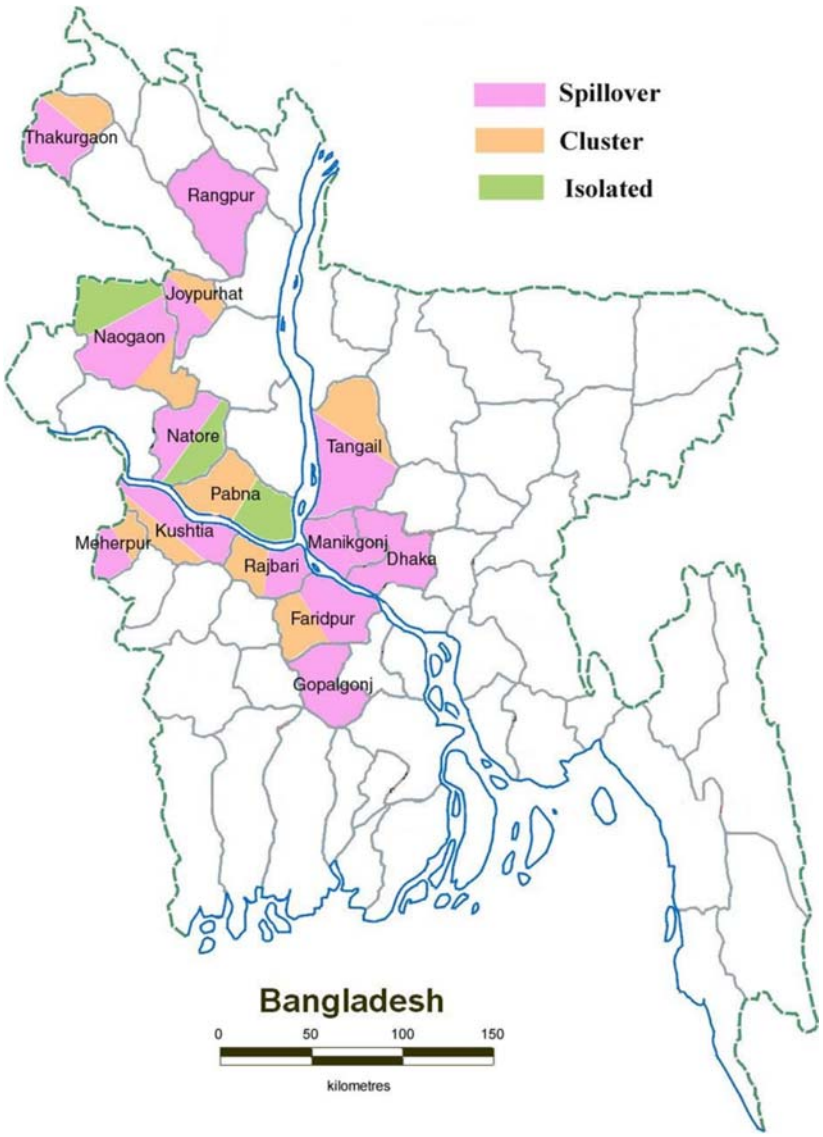


Figure 1. NiV affected areas (i.e., 'NiV belt') in Bangladesh until at the end of 2008.

In Malaysian outbreaks, NiV was transmitted to human via the sick pigs. Since the natural reservoirs of Nipah virus are fruit bats, so the pigs are supposed to become infected by eating fruits partially eaten and

thus contaminated by the fruit bats with their urine and saliva. However, NiV outbreaks in Bangladesh remain devastating because of the mode of transmissions. In the early stage, Nipah virus is supposed to be transmitted through the date palm sap from its natural reservoirs, i.e., fruit bats in addition to the case discussed above. When people drink the contaminated date palm sap, they become infected by the Nipah virus. Once people have been infected by the NiV either by eating the partially eaten and contaminated fruits or by drinking the contaminated raw date palm sap, the virus spreads into human to human because of its serious infectivity. At least one case was reported that a doctor died due to NiV infection while giving the healthcare to a NiV infected patient in hospital. A schematic diagram of possible Nipah virus transmissions in Bangladesh is shown in Figure 3.

Figure 2. NiV affected areas (i.e., 'NiV belt') in Bangladesh at the end of 2013.

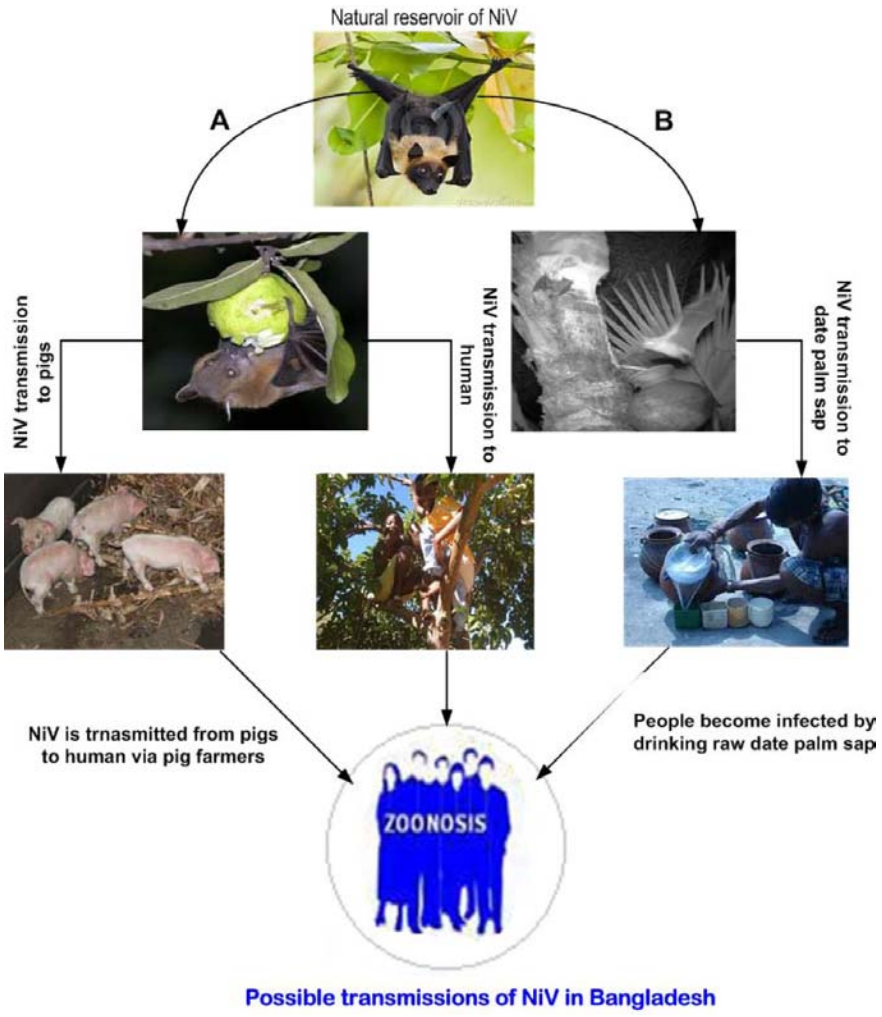


Figure 3. A schematic diagram of possible NiV transmissions in Bangladesh [2].

Table 1. Outbreaks of Nipah virus infections in Bangladesh, 2001-2013 [19]

Outbreaks (Years)	Infected people	No. of deaths	Percentage (%)	Remarks
2001	13	9	69%	–
2002	0	0	0	No outbreak occurs
2003	12	8	67%	–
2004	67	50	75%	–
2005	12	11	92%	–
2006	0	0	0	No outbreak occurs
2007	20	13	65%	–
2008	10	9	90%	–
2009	4	1	25%	–
2010	17	15	88%	–
2011	24	24	100%	–
2012	6	6	100%	–
2013	24	21	87.5%	–
Total	209	167	80%	Average mortality rate

3. Mathematical Model

Mathematical models have become important tools in analyzing the spread and control of infectious diseases. An efficient preventive and control measure of the spread of a life-threatening pathogen mainly depends on an essential understanding the mechanisms of that pathogen. Mathematical models of infectious diseases in human have been used to increase our understanding of these mechanisms and to test hypotheses about effective methods for prevention and control of infectious diseases in humans. The transmission interactions in a population are very complex, so it is difficult to comprehend the large scale dynamics of disease spread. Understanding these interaction characteristics can lead to better approaches to decreasing the transmission of diseases. Mathematical models are used in such comparing, planning

implementing, evaluating, and optimizing various detection, prevention, therapy and control programs. We recall that NiV is a newly detected highly emerging pathogen and no proper drugs and/or vaccines are available yet for it's treatments. So, it is essential for the physicists and biologists to understand the disease mechanisms in the human body in order to find out effective methods for prevention and control.

Human NiV is a *zoonotic* virus and thus transmitted first from animal to human. Once it has been transmitted to human, then it continues to be transmitted through human to human (H2H) by the close contact of infected individuals due to it's highly infectivity. So, the dynamics of Nipah virus (NiV) infections can be described by an *SIR type* infectious disease model in the form of a set of ordinary differential equations (ODEs). Let us suppose that $S(t)$, $I(t)$, and $R(t)$ denote the number of individuals in the susceptible, infectious, and recovered classes at time t. The total population at time t is represented by $N(t) = S(t) + I(t) + R(t)$. The susceptible ($S$) individuals are those able to be infected by the disease parasite. It is assumed that all people are susceptible by born. The infectious (I) individuals are those who are infected and able to transmit the parasite to others and the recovered (R) individuals are those who have recovered and thus are immune or have died from the disease and do not contribute to the transmission of the disease.

3.1. NiV epidemic model

We recall that the basic SIR epidemic models are used to describe rapid outbreaks that occur in a short duration of time (e.g., less than one year). Since the time period is short, no vital dynamics (births and deaths) are considered in this model. From the disease analysis, we see that Nipah outbreaks occur in a very short duration of time (e.g., 3-4 months) and also Nipah is a *zoonotic* virus, such type of disease model was discussed in [2] (see also [1], [6], and [15]). Considering all these as well as the main two transmission routes of Nipah infections in Bangladesh, we take the NiV model as the following ordinary differential equations:

$$\dot{S}(t) = -\beta \frac{S(t)}{N} I(t) - u_1 S(t),$$

$$\dot{I}(t) = \beta \frac{S(t)}{N} I(t) - (\alpha + \gamma) I(t) - u_2 I(t),$$

$$\dot{R}(t) = u_1 S(t) + u_2 I(t) + \gamma I(t), \tag{1}$$

with the initial conditions

$$S(0) = S_0 \geq 0, \quad I(0) = I_0 \geq 0, \quad \text{and } R(0) = R_0 \geq 0. \tag{2}$$

In model (1), β is the incident coefficient representing the average number of adequate contacts (i.e., contacts sufficient for transmission) of a person per unit time, then $\beta \frac{I}{N}$ is the average number of contacts with infected individuals per unit time of one susceptible, and $\beta \frac{I}{N} S$ is the number of new cases per unit time due to the susceptibles. This form of the *horizontal incidence* is called the *standard incidence* (see for details [15]). The simple mass action law ηIS, with η as a *mass action coefficient*, is also sometimes used for the horizontal incidence. In this case, the parameter η has no direct epidemiological interpretation [15], but comparing it with the standard formulation, it shows that $\beta = \eta N$, so that this form implicitly assumes that the contact rate *beta* increases linearly with the population size. However, it is shown in [15] that the *standard incidence* is more realistic for human diseases than the *simple mass action incidence*. The term γI represents the recovery rate with the *recovery coefficient* γ and αI is the disease induced death rate with the coefficient α.

We introduce two additional variables representing controls denoted by $u = (u_1, u_2)$ in the dynamics. As we discussed before, no proper treatment (neither by vaccination nor appropriate drugs) is available for NiV infections till now. The only ways to control the disease and/or

prevent people from being infected by Nipah virus are (i) before outbreak: huge mass and educational campaigns among the people of the risky areas so that they can be motivated from drinking raw date sap and (ii) after outbreak: the family members and relatives of the infected patients as well as the health-care givers (e.g., doctors and nurses) should follow '*social distances*' [26] so that no more H2H infections occur. So our controls $u_1(t)$ measure the effort needed to increase mass and educational campaigns, reducing the effective transmission rate β and $u_2(t)$ measure the effort required social distancing while administering antiviral drug treatment and/or giving health cares to novel the infected individuals. We assume that our control functions are bounded and Lebesgue measurable on the interval $[0, T]$, where T denotes a pre-selected length of time during which these controls are applied. Furthermore, wherever a full effort is being placed on mass campaign or social distancing at time t, we would have that $u_1(t)$ and $u_2(t)$ must be equal to one. Moreover, when no effort is being placed in these controls at time t, then $u_1(t)$ and $u_2(t)$ are equal to zero. Under the above settings, we define the controls taking values in measurable control set

$$U = \{(u_1(t), u_2(t)) : 0 \le u_i(t) \le 1, i = 1, 2, \text{a.e. } t \in [0, T]\}.$$

Our objective is to be chosen as the cost functional

$$\text{Minimize } J(u_1(t), u_2(t)) = I(t) + \int_0^T \frac{1}{2}(B_1 u_1^2 + B_2 u^2(t))dt, \tag{3}$$

where B_1 and B_2 are weight parameters balancing the costs.

From the characteristics of the NiV infections, it is clear that the outbreaks of NiV occur for a short period of time. In Bangladesh, all the previously occurred NiV outbreaks lasted for 2-4 weeks and resulted in a devastating consequences of high mortality. In this regard, the dynamics of NiV infections can appropriately be described by the epidemic model presented in (1).

Table 2. Parameters and constants with clinically approved values [1]

Parameters and constants	Definition of parameters	Clinical values
β	Incidence coefficient	0.75
γ	Recovery rate	0.1
α	Disease induced death rate	0.15
T	Number of days	50
S_0	Initial susceptible population	1000
I_0	Initial infected population	5.0
R_0	Initial recovered population	0
N	Initial population	1005

4. Analysis for Optimal Solution

It is easy to see that our proposed model in (1) can be reformulated in the following optimal control form:

$$(P)\begin{cases} \text{Minimize } l(x(t)) + \displaystyle\int_0^T L(x(t),\, u(t))dt \\ \text{subject to} \\ \quad \dot{x}(t) = f(x(t)) + g(x(t))u(t) \text{ a.e. } t, \\ \quad u(t) \in [0,\, 1] \text{ a.e. } t, \\ \quad x(0) = x_0, \\ \quad x(T) \in E, \end{cases}$$

where $E \subset \mathbb{R}$ and the functions are

$$x(t) = (S(t),\, I(t)),$$

$$l(x(t)) = I(t),$$

$$L(x,\, u) = \frac{1}{2}\left(B_1 u_1^2(t) + B_2 u_2^2(t)\right),$$

$$f(x) = \left(\beta \frac{S(t)}{N} I(t),\ \beta \frac{S(t)}{N} I(t) - (\alpha + \gamma) I(t)\right),$$

$$g(x) = \begin{pmatrix} -S(t) & 0 \\ 0 & -I(t) \end{pmatrix} \text{ and } u(t) = (u_1(t),\ u_2(t)).$$

We define the Hamiltonian

$$H(x,\ u,\ p,\ \lambda) = p \cdot f(x) + p \cdot g(x)u - \lambda L(x,\ u).$$

The necessary conditions of optimality for optimal control problem (P) can be obtained by applying the well-known Pontryagin Maximum Principle for optimal control problem. In vein of [28], the necessary conditions give closed forms for the controls (taking into account the control constraints) of our problem. It is worth mentioning that our cost is convex in u and the dynamics are linear in u. In such case, the optimal solution of our model is guaranteed by [13].

Suppose that $(x^*,\ u^*)$ is the optimal solution of the above problem. The maximum principle in [28] asserts the existence of an absolutely continuous function p and a scalar $\lambda \geq 0$ such that

(i) $\|p\|_\infty + \lambda > 0,$

(ii) $-\dot{p}(t) = p(t) \cdot f_x(x^*(t)) + p(t) \cdot g_x(x^*(t))u^*(t) - \lambda L_x(x^*(t),\ u^*(t)),$

(iii) $\forall u \in U,\ p(t) \cdot g(x^*(t))u^*(t) - \lambda L(x^*(t),\ u^*(t)) \geq p(t) \cdot g(x^*(t))u(t) - \lambda L(x^*(t),\ u(t))$a.e.,

together with the transversality condition $p(T) = (0,\ w),\ w \in \mathbb{R}.$

Let us consider that $p(t) = (p_s,\ p_i)$. Then we deduce from (iii) an explicit characterization of the optimal control pair $(u_1^*(t),\ u_2^*(t))$ given in terms of the multipliers p as

$$\left(\min\left\{\max\left\{0, \frac{-p_s S}{B_1}\right\}, 1\right\}, \min\left\{\max\left\{0, \frac{-p_i I}{B_2}\right\}, 1\right\}\right).$$

We note that the total population is $N(t) = S(t) + I(t) + R(t)$ and the state variable R does not appear in other differential equations. Since it can be obtained by $R(t) = N(t) - S(t) - I(t)$, so this is disregarded while solving the optimality systems.

5. Numerical Results

We performed numerical simulations to obtain the optimal control schedules for our model in different scenarios. To do these simulations, we used the nonlinear optimal control solver "ICLOCS", version 0.1b [12]. "ICLOCS" is an optimal control interface, implemented in Matlab, for solving the optimal control problems with general path and boundary constraints and free or fixed final time. "ICLOCS" uses the "IPOPT" solver, which is an open-source software package for large-scale nonlinear optimization [29].

Considering a time interval of 50 days, a time-grid with 1000 nodes was created, that is, for $t \in [0, 50]$, we get $\Delta t = 0.05$. All initial values and the parameters used in our analysis are presented in Table 2, which are similar as in [2] (see also [1] and [6]). Since we used a direct method and, consequently, a iterative approach, we imposed an acceptable convergence tolerance at each step of $\varepsilon_{\text{rel}} = 10^{-9}$.

For the convenience of comparing the results, we run the program taking both '*without control*' and '*with control*' into account, but the same cost as in (3). We first solve the model when no control measures were initiated for the treatment. The result obtained in this case is presented in Figure 4.

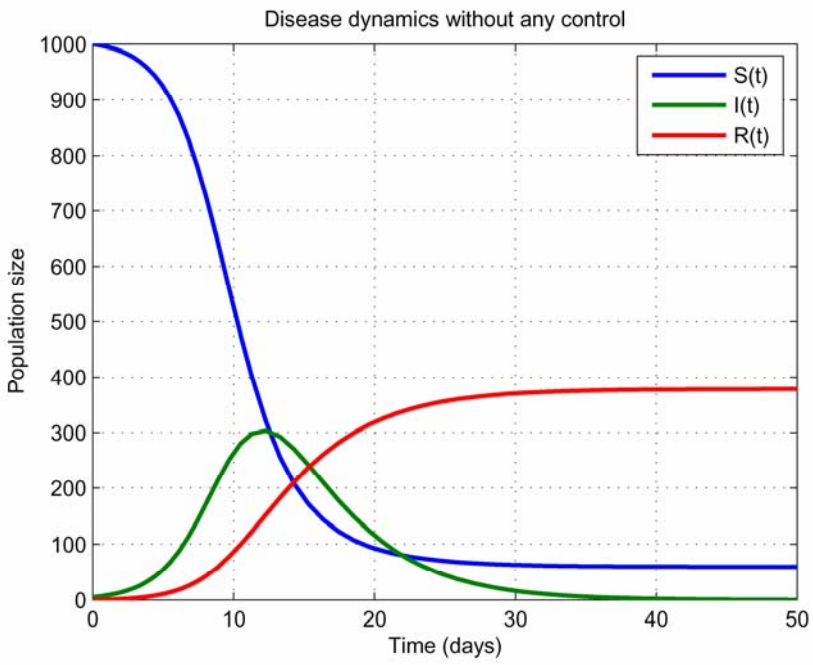
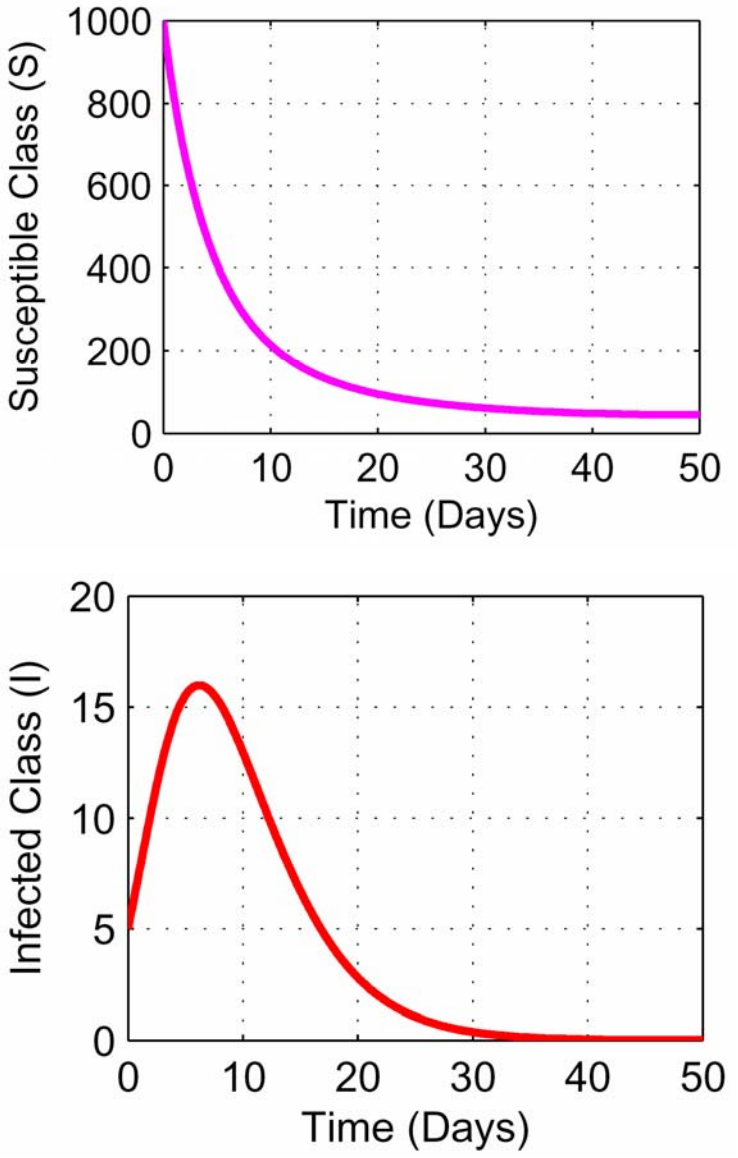


Figure 4. The disease behaviour of different population sizes in absence of control measures.

From Figure 4, we observe that the infectious class first increases from an initial state I_0 near zero, reaches a peak, and then decreases toward zero over a finite interval of time. The susceptible class $S(t)$ always decreases, but the final susceptible class is positive, this is because of the decreasing nature of the function S over time t. On the other hand, according to the basic reproductive number $R_0 > 1$, the figure of infectious class says that such epidemic occurs and continues in a certain region. However, the epidemic can be prevented if it is possible to satisfy the condition $R_0 S(0) < 1$. Thus, if the initial susceptible fraction has been reduced to less than $\frac{1}{R_0}$, for example, by immunization, then an epidemic can be prevented and this can be achieved by preventing the susceptible people from being infected by mass campaigns and social distances.

Next, we solve for the optimality systems of our proposed model when the two control measures were introduced to prevent the infections. We run the program considering the weight parameters $B_1 = 0.1$ and $B_2 = 0.5$. The simulation results for the optimal states (i.e., individual population class) and the controls are shown in Figure 5.



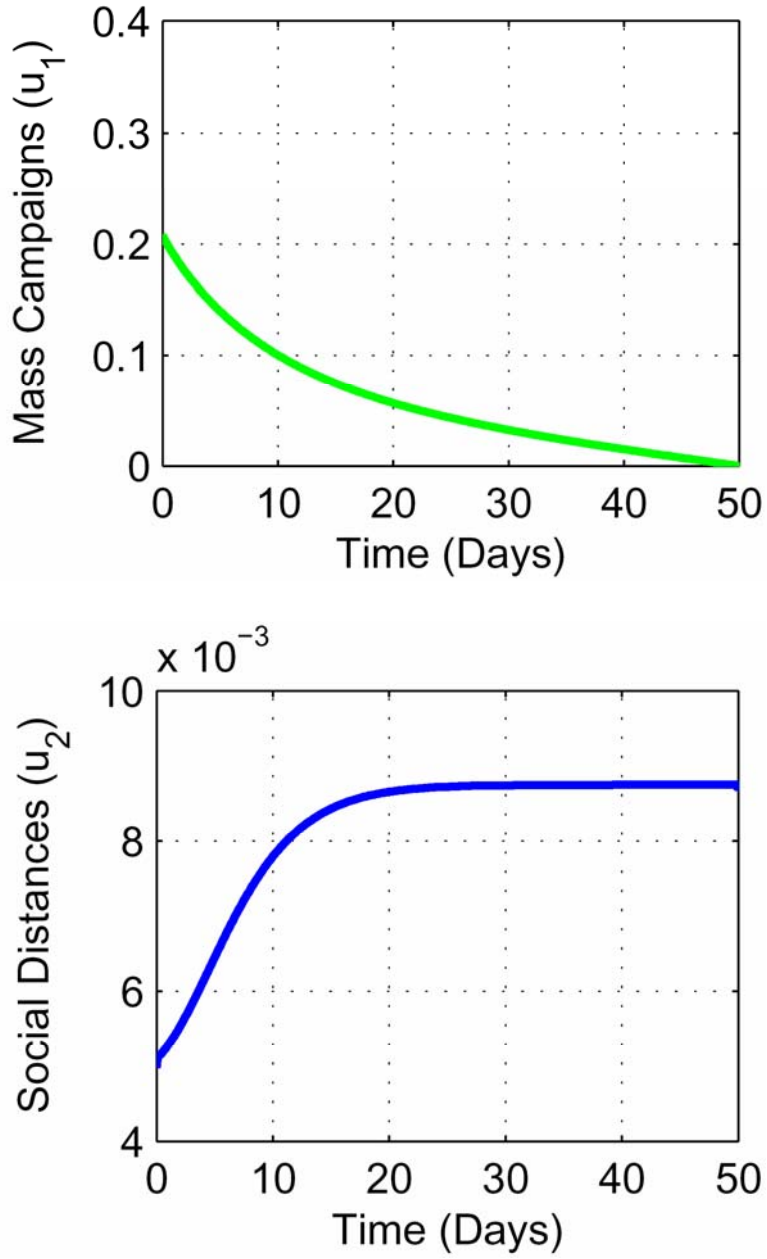
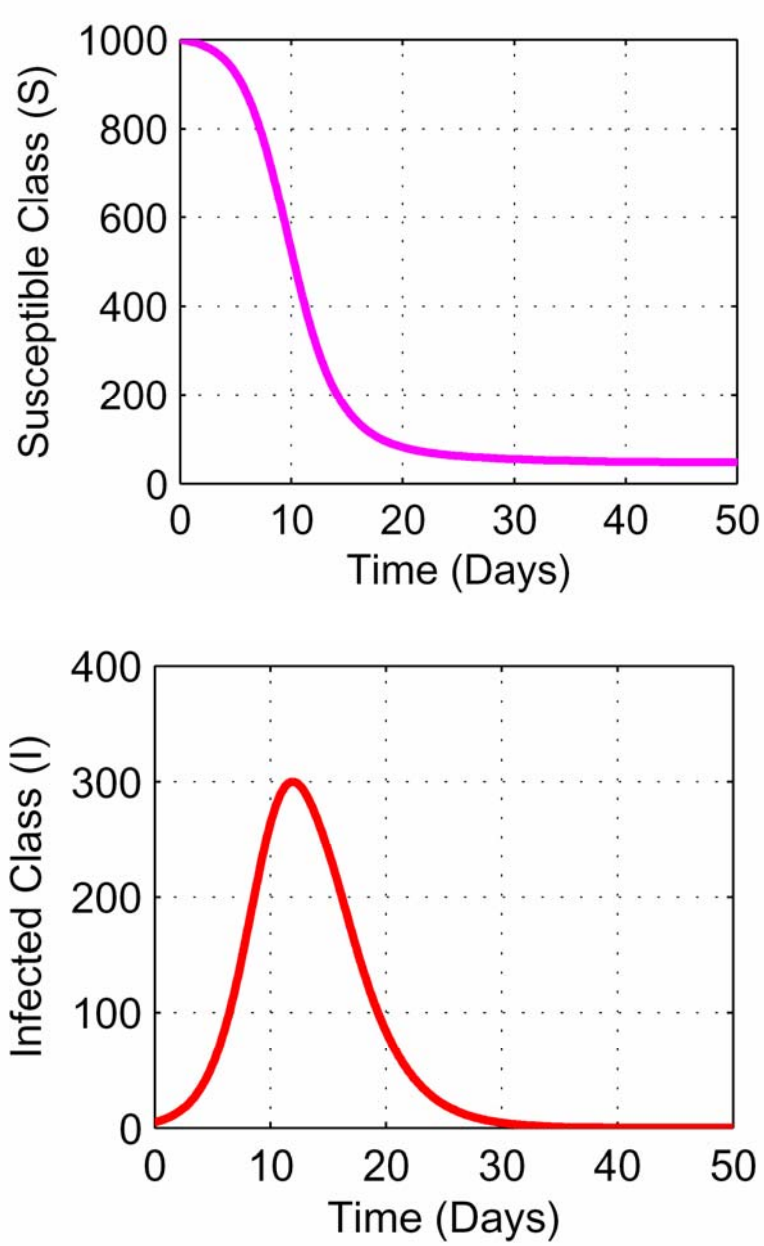


Figure 5. Optimal trajectories for the susceptible and infectious classes and optimal controls: Mass campaigns and social distances with weight parameters $B_1 = 0.1$ and $B_2 = 0.5$.

Now we take the weight parameters $B_1 = 0.5$ and $B_2 = 0.1$ and we run the optimality systems. The simulation results for the optimal states (i.e., individual population class) and the controls are shown in Figure 6.



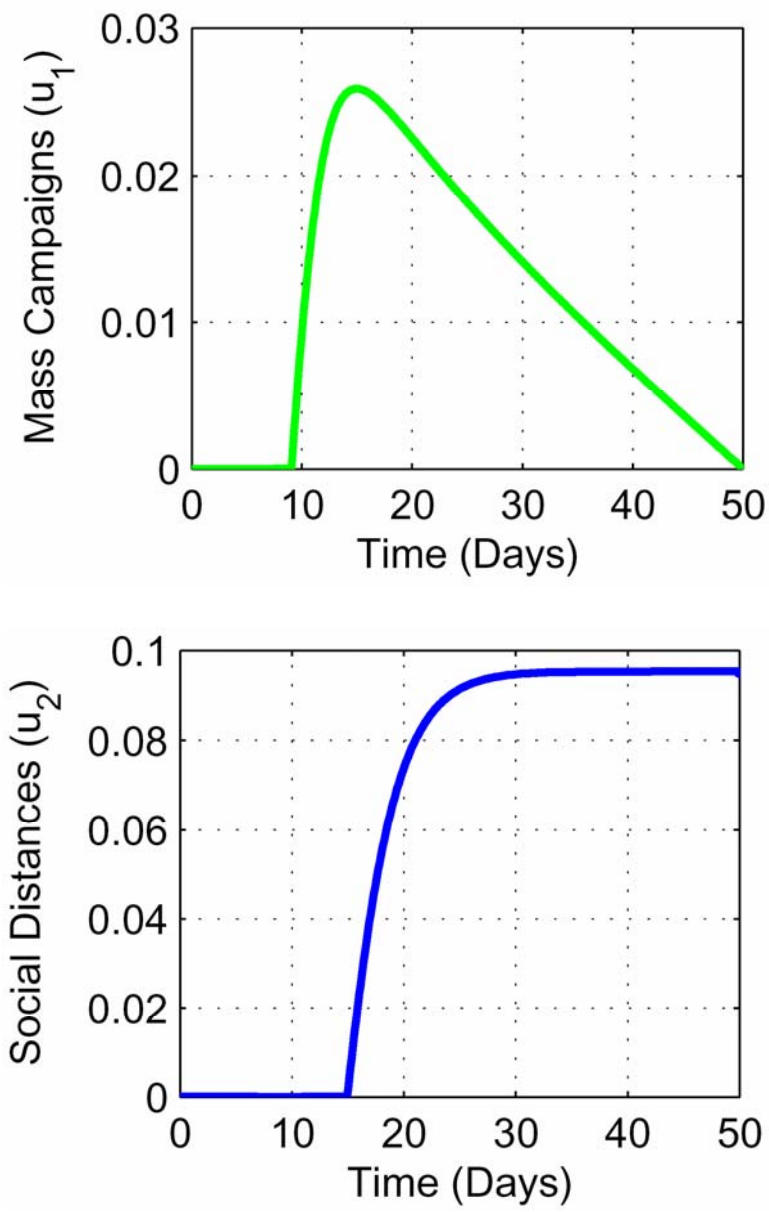
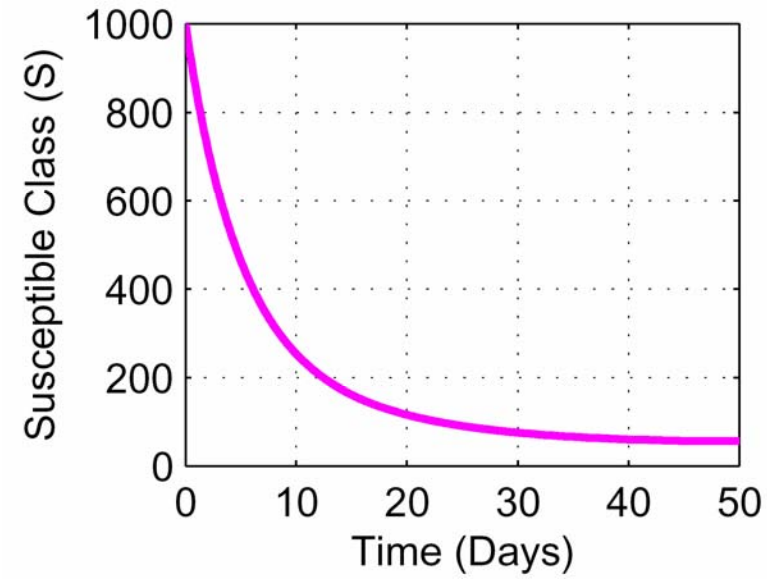
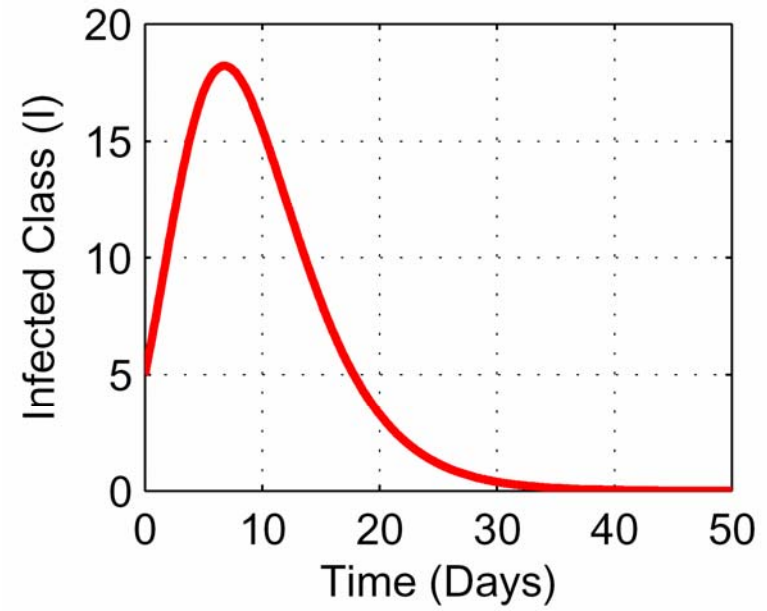


Figure 6. Optimal trajectories for susceptible and infectious classes and optimal controls: Mass campaigns and social distances with weight parameters $B_1 = 0.5$ and $B_2 = 0.1$.

We also run the program considering the weight parameters $B_1 = B_2 = 0.1$ and $B_1 = B_2 = 0.5$ and the simulation results for the optimal states (i.e., individual population class) and the controls are shown in Figures 7 and 8, respectively.





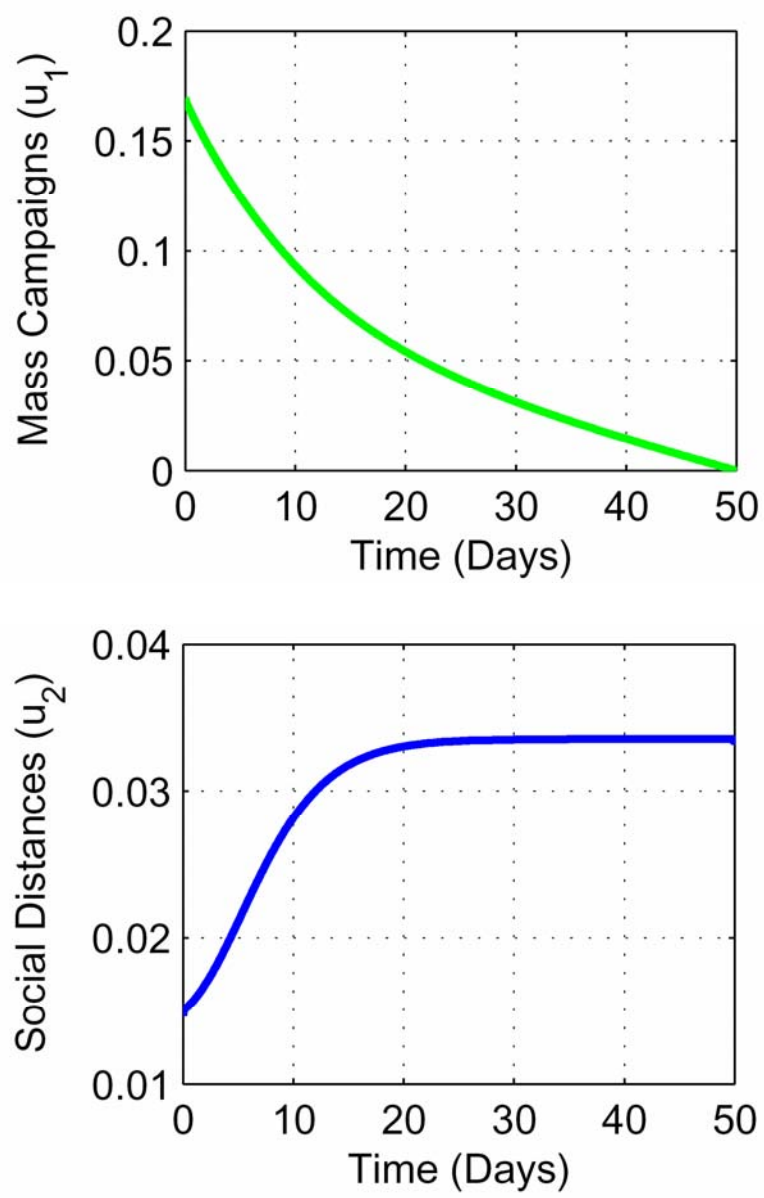
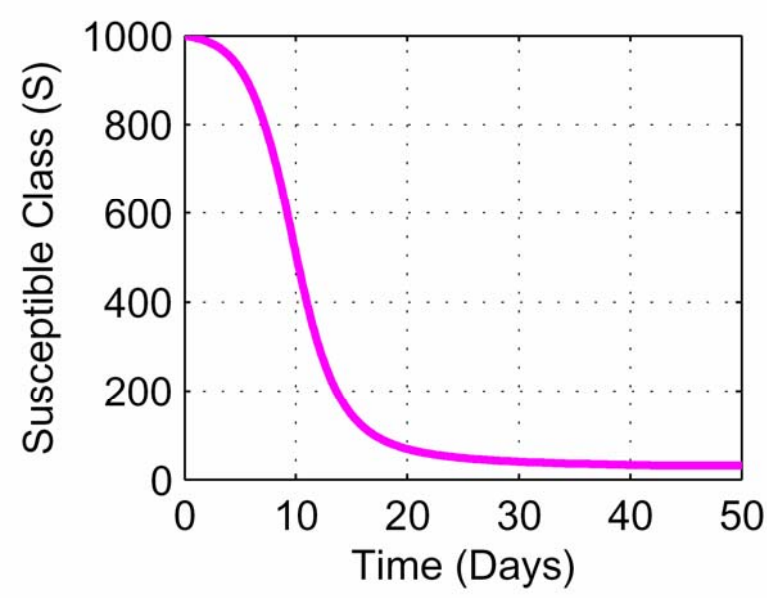
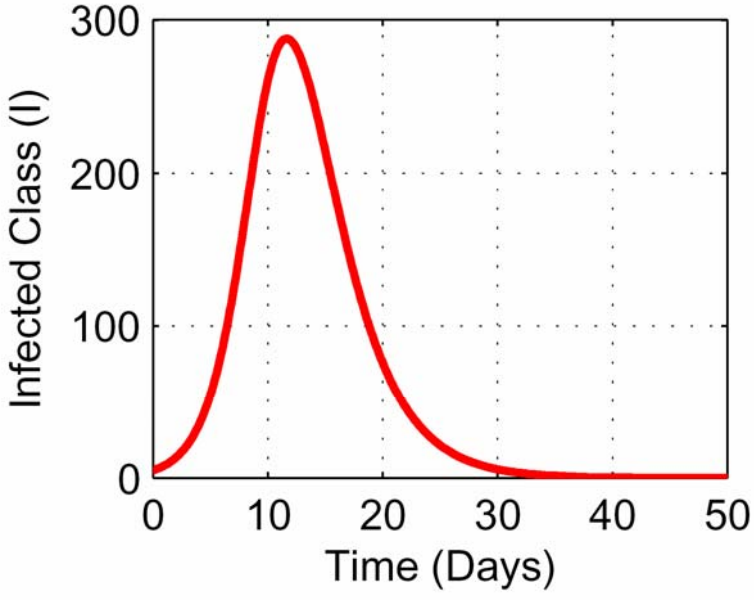


Figure 7. Optimal trajectories for susceptible and infectious classes and optimal controls: Mass campaigns and social distances with weight parameters $B_1 = 0.1$ and $B_2 = 0.1$.


Susceptible Class (S)
1000
800
600
400
200
0
0
10
20
30
40
50
Time (Days)


Infected Class (I)
300
200
100
0
0
10
20
30
40
50
Time (Days)

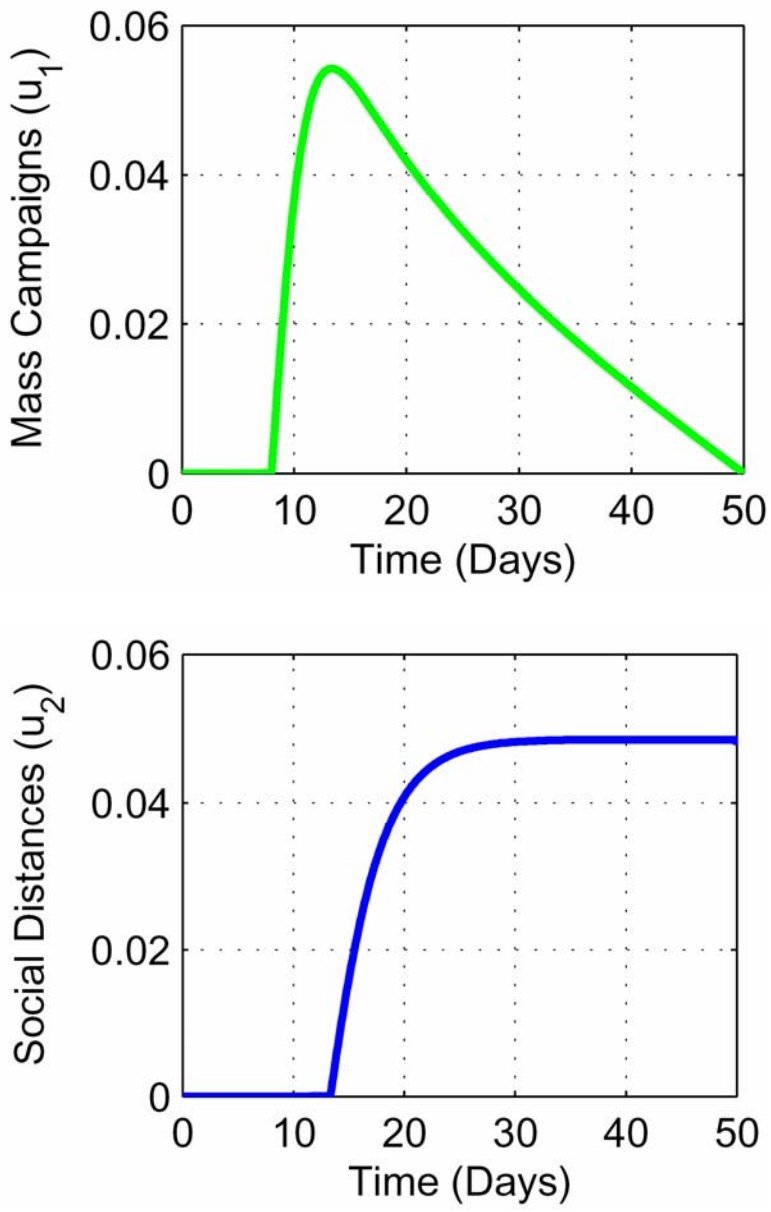


Figure 8. Optimal trajectories for susceptible and infectious classes and optimal controls: Mass campaigns and social distances with weight parameters $B_1 = 0.5$ and $B_2 = 0.5$.

From our above analysis and figures, it is easy to compare that the control induced model gives a better preventive and control strategy for the spread of diseases transmissions and thus control the infections. It is worth noting that the weight parameters balancing the cost play important role in this control strategy. Less weight value of the respective control function gives the maximum effective control strategy.

6. Conclusion

NiV infection is a highly pathogenic epidemic disease in the south-east Asian countries, mostly in Bangladesh. It is suspected that if the present trends of NiV outbreaks in Bangladesh continue, then it is not surprising that in our globally connected world, humanity could face its most devastating pandemic. As there is no proper treatment with effective drugs and/or vaccines available until to date, 'mass awareness' as well as 'social distances' are the only ways to prevent and control people from being infected by the NiV. Such a control strategy is discussed via a mathematical model of NiV infections in terms of ordinary differential equations. In absence of proper drugs available, our proposed results can be of help in controlling and preventing the diseases.

Conflict of Interest

The author has no conflict of interest that might affect the publication of this manuscript.

Acknowledgement

The author would like to thank Mathematics Discipline of Khulna University, Bangladesh for all sorts of logistic supports while preparing this manuscript.

References

[1] L. J. S. Allen, V. L. Brown, C. B. Jonsson, S. L. Klein, S. M. Laverty, K. Magwedere, J. C. Owen and P. Van Den Driessche, Mathematical modeling of viral zoonoses in wildlife, Natural Resource Modeling 25(1) (2012), 5-51.

[2] M. H. A. Biswas, Model and control strategy of the deadly Nipah virus (NiV) infections in Bangladesh, Research & Reviews in Biosciences 6(12) (2012), 370-377.

[3] M. H. A. Biswas, AIDS epidemic worldwide and the millennium development strategies: A light for lives, HIV and AIDS Review 11(4) (2012), 87-94.

[4] M. H. A. Biswas, Optimal chemotherapeutic strategy for HIV infections-state constrained case, Proceedings of the 1st PhD Students Conference in Electrical and Computer Engineering, Department of Electrical and Computer Engineering, Faculty of engineering, University of Porto, Portugal on June 28-29, 2012.

[5] M. H. A. Biswas, L. T. Paiva and M. D. R. de Pinho, A SEIR model for control of infectious diseases with constraints, Mathematical Biosciences and Engineering 11(4) (2014), 761-784.

[6] M. H. A. Biswas, M. Wahiduzzaman and R. Sultana, Outbreaks of Deadly Nipah Virus (NiV) Infections in Bangladesh: An Analysis of Mathematical Model, Proceedings of International Conference on Engineering Research, Innovation and Education (ICERIE 2013) (2013), 309-315.

[7] R. Breban, J. M. Drake, D. E. Stallknecht and P. Rohani, The role of environmental transmission in recurrent Avian Influenza epidemics, PLoS Comp. Biol. 5(4) (2009), 1-11.

[8] H. T. Chong, S. A. Bakar and C. T. Tan (Edts.), Proceedings of the Nipah virus colloquium 2008, University of Malaya and related articles, Neurology Asia 14 (2009), 45-47.

[9] H. T. Chong, M. J. Hossain and C. T. Tan, Differences in epidemiologic and clinical features of Nipah virus encephalitis between the Malaysian and Bangladesh outbreaks, Neurology Asia 13 (2008), 23-26.

[10] K. B. Chua, W. J. Bellini and P. A. Rota et al., Nipah virus: A recently emergent deadly paramyxovirus, Science 288(5470) (2000), 1432-1435.

[11] K. B. Chua, Nipah virus outbreak in Malaysia, J. Clin. Virol. Apr. 26(3) (2003), 265-275.

[12] P. Falugi, E. Kerrigan and E. Van Wyk, Imperial College London Optimal Control Software User Guide (ICLOCS), Department of Electrical and Electronic Engineering, Imperial College London, London, England, UK, 2010.

[13] W. H. Fleming and R. W. Rishel, Deterministic and Stochastic Optimal Control, Applications of Mathematics No. 1, Springer Verlag, New York, 1975.

[14] T. W. Geisbert, K. M. Daddario-DiCaprio, A. C. Hickey, M. A. Smith, Y.-P. Chan et al., Development of an acute and highly pathogenic nonhuman primate model of Nipah virus infection, PLoS ONE 5(5) (2010), 1-12.

[15] H. W. Hethcote, The mathematics of infectious diseases, SIAM Review 42(4) (2000), 599-653.

[16] M. K. Lo, L. Lowe, K. B. Hummel, H. M. S. Sazzad, E. S. Gurley, M. J. Hossain, S. P. Luby, D. M. Miller, J. A. Comer, P. E. Rollin, W. J. Bellini and P. A. Rota, Characterization of Nipah virus from outbreaks in Bangladesh, 2008-2010, Emerging Infectious Diseases 18(2) (2012), 248-255.

[17] S. P. Luby, The pandemic potential of Nipah virus, Antiviral Research 100 (2013), 38-43.

[18] S. P. Luby, E. S. Gurley and M. J. Hossain, Transmission of human infection with Nipah virus, Clin. Infect. Dis. 49(11) (2009), 1743-1748.

[19] National Guideline for Management, Prevention and Control of Nipah Virus Infection including Encephalitis, Directorate General of Health Services, Ministry of Health & Family Welfare, Government of the People's Republic of Bangladesh, 2011.

[20] R. M. Neilan and S. Lenhart, An introduction to optimal control with an application in disease modeling, DIMACS Series in Discrete Mathematics 75 (2010), 67-81.

[21] Nipah outbreak in Lalmonirhat district, 2011, ICDDR,B, Health and Science Bulletin 9(2) (2011), 13-18.

[22] Nipah outbreak in Faridpur district, Bangladesh, 2010, ICDDR,B, Health and Science Bulletin 8(2) (2010), 6-11.

[23] Nipah Virus Infections, WHO Report, Asia-Pacific Region, World Health Organization, 2008.

[24] N. I. Paton, Y. S. Leo and S. R. Zaki et al., Outbreak of Nipah-virus infection among abattoir workers in Singapore, Lancet 354(9186) (1999), 1253-1256.

[25] U. D. Parashar, L. M. Sunn and F. Ong et al., Case-control study of risk factors for human infection with a new zoonotic paramyxovirus, Nipah virus, during a 1998-1999 outbreak of severe encephalitis in Malaysia, The Journal of Infectious Diseases 181(5) (2000), 1755-1759.

[26] O. Prosper, O. Saucedo, D. Thompson, G. Torres-Garcia, X. Wang and C. Castillo-Chavez, Modeling control strategies for concurrent epidemics of seasonal and pandemic H1N1 influenza, Mathematical Biosciences and Engineering 8(1) (2011), 141-170.

[27] R. Stone, Breaking the Chain in Bangladesh, Science 331 (2011), 1128-1131.

[28] R. Vinter, Optimal Control, Birkhäuser, Boston, 2000.

[29] Andreas Wächter and Lorenz T. Biegler, On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming, Mathematical Programming 106 (2006), 25-57.

[30] T. T. Yusuf and F. Benyah, Optimal control of vaccination and treatment for an SIR epidemiological model, World Journal of Modelling and Simulation 8(3) (2012), 194-204.

■